Paramount Domination in Bipolar Fuzzy Graphs

V. Mohanaselvi, S. Sivamani

Abstract— A dominating set D $\subseteq$ V of a bipolar fuzzy graph B_G is said to be paramount dominating set pad-set if V/D is not a dominating set of B_G. The minimum cardinality of a pad- set is called paramount domination number of B_G and it is denoted by $\gamma_{pa}(B_G)$.In this paper some results on paramount domination number are obtained. In this paper, we studied this parameter for connected non complete fuzzy graphs.

Key words— Bipolar fuzzy graph (BFG), strong edge, dominating set, domination number, paramount dominating set, minimal paramount dominating set, paramount domination number.
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1 Introduction

IN 1965, Zadeh [9] introduced the notion of fuzzy subset of a set as a method of presenting uncertainty.In 1962, the study of dominating sets in graphs was begun by Ore and Berge, the domination number, independent domination number are introduced by Cockayne and Hedetniemi in 1977.

Rosenfeld introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness in 1975, whose basic idea was introduced by Kauffman in 1973.In 1988, A.Somasundram and S.Somasundram [7] discussed domination in fuzzy graph. They defined domination using effective edges in fuzzy graph.

In 1994, Zhang [10] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are extension of fuzzy sets whose range of membership degree is [−1, 1]. In 2011, Akram [1] introduced the concept of bipolar fuzzy graphs and defined different operations on it. Bipolar fuzzy graph theory is now growing and expanding its applications.

Recently in 2013, M. G. Karunambigai, S. Sivasankar, M. Akram, K. Palanivel [2], introduce the concept of domination in bipolar fuzzy graphs. In 2015 V.Mohanaselvi, S.Sivamani [5][6] introduce Perfect domination in bipolar fuzzy graphs and Strong (weak) domination in bipolar fuzzy graphs. The concept of The Maximal domination number of a graph was introduced by V,R. Kulli and B.Janakiraman[3] in 1997. Complementary nil domination in a crisp graph was introduced by T.Tamizh Chelvam and S.Robinson Chellathurai [8] in 2009.Complementary nil domination in a fuzzy graph was introduced by Mohamed Ismayil, Ismail Mohideen [4] in 2014.

In this paper, our aim is introduce and study the theory of paramount domination in BFG. Throughout this paper, a graph B_G = (V, E) will denote a bipolar fuzzy graph without loops. Also in this paper only connected non complete fuzzy graphs are considered. The following definitions are used in this paper.

Let X be a non empty set. A *bipolar fuzzy set* M in X is an object having the form M = {(x, $\mu^+(x)$, $\mu^-(x)$) | x∈ X} where μ^+: X → [0, 1] and μ^-: X → [−1, 0] are mapping.

A bipolar fuzzy graph (BFG) is of the form B_G = (V, E) where (i)V= {v_1, v_2 ...vn} such that μ_1^+ : V→ [0, 1] and μ_1^- : V → [−1, 0] (ii) E⊂V× V where μ_2^+: V× V→ [0, 1] and μ_2^-: V × V→ [−1, 0] such that $\mu_{2ij}^+ = \mu_2^+(v_i, v_j) \le \min(\mu_1^+(v_i), \mu_1^+(v_j))$ and $\mu_{2ij}^- = \mu_2^-(v_i, v_j) \ge \max(\mu_1^-(v_i), \mu_1^-(v_j))$ $\forall$ $(v_i, v_j) \in E$

The order p and size q of an BFG, B_G = (V, E) are defined to be $p = \sum_{v_i \in V}\left(\frac{1+\mu_1^+(vi)+\mu_1^-(vi)}{2}\right)$ and

$$q = \sum_{(v_i,v_j)\in E}\left(\frac{1+\mu_2^+(vi,vj)+\mu_2^-(vi,vj)}{2}\right)$$

An edge e = (x, y) of B_G is called an effective edge (strong edge) if $\mu_2^+(x, y) = \min(\mu_1^+(x), \mu_1^+(y))$ & $\mu_2^-(x, y) = \max(\mu_1^-(x), \mu_1^-(y))$. The vertices u, v are called adjacent in G.

Let B_G be a BFG on V. Let u, v ∈ V, we say that u *dominates* v in G if there exists an effective edge between them. A subset D of V is called a *dominating set* in G if for every v ∈V− S, there exists u∈S such that u dominates v.

A dominating set D is called minimal, if for every u ∈ V−D, the set D-{u} is not dominating set in B_G. The minimum cardinality among all minimal dominating set is called domination number, denoted by $\gamma(B_G)$.

2 Paramount domination in biploar fuzzy graphs

2.1 Defintions:

Definition 2.1: A dominating set D⊆V in a B_G is called a paramount dominating set (or simply called pad-set) of a bipolar fuzzy graph G if V-D is not a dominating set.

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Definition 2.2: A paramount dominating set D of B_G is said to be minimal paramount dominating set if for every u∈ V−D, the set D-{u} is not a paramount dominating set in BG .

Definition 2.3: The minimum scalar cardinality taken over all pad-set is called a paramount domination number and is denoted by the symbol $\gamma_{pa}(B_G)$, the corresponding minimum pad-set is denoted by $\gamma_{pa}(B_G)$-set.
The maximum scalar cardinality taken over all minimal pad-set is called a upper paramount domination number and is denoted by the symbol $\Gamma_{pa}(B_G)$.

Definition 2.4: Let B_G = (V, E) be a BFG, D⊆V. A vertex u∈D is said to be an enclave of D if for all v∈V-D μ_2^+(u, v) < min (μ_1^+ (u), μ_1^+ (v)) and μ_2^- (u,v) > max (μ_1^- (u), μ_1^- (v))

Example 2.5

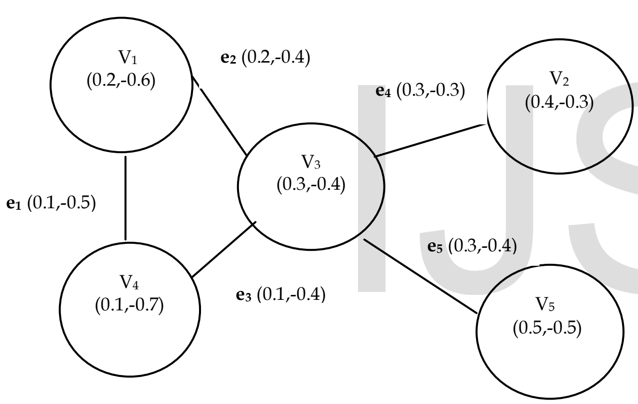

Figure: 1

- For the figure: 1 the following results are obtained.
- p = 2 and q=2
- $\delta_N = 0.45 \quad \Delta_N = 1.55$
- Min | v_i | = | v_4 |= 0.20
- Max | v_i | = | v_2 |= 0.55
- Minimal dominating set: D = { v_3 }; $\gamma(B_G) = 0.45$
- Enclaves of D: v_1, v_2, v_4, v_5
- Minimal pad sets: D_1= { v_3, v_4 }, D_2 = { v_2, v_3 } , D_3 = { v_3, v_1 }, D_4 = { v_3, v_4 }

Remarks 2.6: For any bipolar fuzzy graph BG,
(1) Every super set of a pad-set is also a pad-set.
(2) Complement of a pad-set is not a pad-set.
(3) Complement of a dominating-set is not a pad-set.
(4) A pad-set need not be unique.
(5) $\gamma(B_G) \le \gamma_{pa}(B_G)$

2.2 Main Results

Theorem 2.1: *A dominating set D is a pad-set if and only if it contains at least one enclave.*

Proof: Let D be a pad-set of B_G. Then V -D is not a dominating set which implies that there exists a vertex u∈D such that u is not dominating any element in V-D. Therefore u is an enclave of D. Hence D contains at least one enclave.

Conversely, suppose the dominating set D contains enclaves. Without loss of generality let us take u be the enclave of D. That is μ_2^+(u, v) < min (μ_1^+ (u), μ_1^+ (v)) and μ_2^- (u, v) > max (μ_1^- (u), μ_1^- (v)) for all v ∈V -D. Hence V - D is not a dominating set. Hence the dominating set D is a pad-set.

Theorem 2.2: *If D is a pad-set of B_G, then there is a vertex u ∈D such that D-{u} is a dominating set.*

Proof: Let D be a pad-set of B_G. By theorem 2.1, every pad-set contains at least one enclave in D. Let u∈D be an enclave of D. That is μ_2^+(u, v) < min (μ_1^+ (u), μ_1^+ (v)) and μ_2^- (u, v) > max (μ_1^- (u), μ_1^- (v)) for all v ∈V –D.
Since B_G is a connected fuzzy graph, there exists at least a vertex w∈D such that u, w are adjacent vertices. Hence D-{u} is a dominating set.

Corollary 2.3: *If D is a pad-set of B_G, then there exist n enclaves say $u_1, u_2, u_3, u_4,..u_n$ in D such that D-{ $u_1, u_2, u_3, u_4,..u_n$ } is a dominating set, not a pad set.*

Theorem 2.4: *A pad-set of B_G is not a singleton.*

Proof: Let D be a pad-set of B_G. By theorem 2.1, every pad-set contains at least one enclave in D. Let u∈D be an enclave of D. Suppose D contains only one vertex u, shows it must be isolated in B_G, which is a contradiction to connectedness. Hence pad-set contains more than one vertex.

Theorem 2.5: *Let B_G be a bipolar fuzzy graph and D be a pad-set of B_G. If u and v are two enclaves of D, then*
N[u]∩N[v] ≠ ϕ and u and v are adjacent.

Proof: Let D be a pad-set of B_G and let u and v are two enclaves of D. Suppose N[u]∩N[v] = ϕ. Then u is an enclave of D- N(v). Therefore D-N(v) is a pad-set of B_G and |D-N(v)| < |D| , Which is a contradiction to the minimality of D. Hence N[u]∩N[v] ≠ ϕ.
Suppose u and v are non-adjacent. Then u ∉N(v) and so u is an enclave of D-{v}. Hence D-{v} is a pad-set, which is a contradiction to minimality of D. Hence u and v are adjacent.

Theorem 2.6: *A pad-set of B_G is not independent.*

Proof: Let B_G be a bipolar fuzzy graph. Suppose a pad-set D of B_G is independent. Then D is a minimal dominating set, V-D is a dominating set. Hence D is not a pad-set, which is a contradiction.

Theorem 2.7: *A pad-set of B_G is minimal if and only if for each $u \in D$ at least one of the following conditions is satisfied*

(i) $N(u) \cap D = \phi$.

(ii) there exists $v \in$ V-D such that $N(v) \cap D = \{u\}$.

(iii)V- (D –{u}) is a dominating set of B_G.

Proof: Let D be a minimal pad-set of B_G. Suppose, if there exists $u \in D$ such that u does not satisfy any conditions (i) and (ii). Then by a theorem [0], D is not a minimal dominating set. Hence the proper subset D-{u}is a dominating set. By our assumption on (iii) V- (D –{u}) is not a dominating set of B_G. Hence D-{u}is a pad-set, which is a contradiction to the minimality of the pad-set D.

Conversely, let D is a pad-set and for each $u \in D$ at least one of the three conditions holds. Suppose D is not minimal, then there exists a vertex $u \in D$ such that D-{u} is a pad-set. As D-{u} is a pad-set, u is adjacent to at least one vertex in D-{u}, condition (i) does not hold. Also D-{u} is a dominating set, every vertex $v \in$ V-(D-{u}) is adjacent to at least one vertex in D-{u}. That is $N(v) \cap D = \{u\}$, condition (ii) does not hold. As D-{u} is a pad-set, V-(D-{u}) is not a dominating set, condition (iii) does not hold , which is a contradiction to our assumption. Hence D is a minimal pad-set of B_G.

Theorem 2.8: *For any bipolar fuzzy graph B_G every $\gamma_{pa}(B_G)$-set of B_G intersects with every $\gamma(B_G)$-set of B_G.*

Proof: Let D be a $\gamma_{pa}(B_G)$-set and D_1 be a $\gamma(B_G)$-set of B_G. Suppose $D \cap D_1 = \phi$, then $D_1 \subseteq$ V –D, V-D contains a dominating set D_1. Therefore V-D, a super set of D_1, is a dominating set, which is a contradiction.

Corollary 2.9: *For any bipolar fuzzy graph B_G any two $\gamma_{pa}(B_G)$-sets are intersects.*

Theorem 2.10: *For any bipolar fuzzy graph B_G if $\gamma(B_G) = \frac{p}{2}$, then $\gamma_{pa}(B_G) = \frac{p}{2} + \text{Min}\,|v_i|$*

Proof: Let B_G be a bipolar fuzzy graph and let D be $\gamma(B_G)$-set of B_G with $\gamma(B_G) = \frac{p}{2}$. Hence V-D is also $\gamma(B_G)$-set of B_G with $|V - D| = \frac{p}{2}$. Choose a vertex u $\in$ V-D with $|u|$ = $\text{Min}\,|v_i|$. Hence V-D-{u} is not a dominating set. But $D \cup \{u\}$ is a pad-set.

Hence $\gamma_{pa}(B_G) = |D \cup \{u\}| = \frac{p}{2} + \text{Min}\,|v_i|$.

Remark: 2.11: *The converse of theorem 2.11 is not true.*

Theorem 2.12: *For any bipolar fuzzy graph B_G, $\Gamma(B_G) + \gamma_{pa}(B_G) \leq$ p+ Max $|v_i|$*

Proof: Let D be a $\Gamma(B_G)$-set of B_G, then there exists a u $\in$ D such that D-{u}is not a dominating set of B_G. Since D is a minimal dominating set, V -D is a dominating set and [(V -D) $\cup$ {u}] is also a dominating set. Hence [V-(D-{u}] is a pad set. Therefore $\gamma_{pa}(B_G) \leq |(V - D) \cup \{u\}| = p - \Gamma(B_G) + \text{Max}\,|v_i|$. Hence $\Gamma(B_G) + \gamma_{pa}(B_G) \leq p + \text{Max}\,|v_i|$.

Theorem 2.13: *For any bipolar fuzzy graph B_G, $p - q + \delta_N \leq \gamma_{pa}(B_G)$*

Proof: Let B_G be a bipolar fuzzy graph and let D be $\gamma_{pa}(B_G)$-set of B_G. Then there exist a vertex $u \in D$ adjacent only to vertices of D. Thus $q \geq |V - D| + \deg u \geq |V - D| + \delta$. Hence the result.

Theorem 2.14: *For any bipolar fuzzy graph B_G, $\gamma_{pa}(B_G) \leq \gamma(B_G) + \delta_N$*

Proof: Let B_G be a bipolar fuzzy graph and let D be $\gamma(B_G)$-set of B_G, v be a vertex of minimum degree. Then either $v \in D$ or some vertex u adjacent to $v \in D$. Thus $D \cup N[v]$ is a maximal dominating set of B_G. Hence the result.

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